Homework 7

Row Reduced Form

Exercise 1. rref and Dimensions.

Try to spot linear dependence relations between either the columns or the rows of the following matrices. Deduce their rank and the dimension of their kernel. Then reduce the matrices all the way to R, their row reduced echelon form, give a basis for the nullspace, and then maybe check (in julia/matlab/octave) with `rref()` that you got it right.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \qquad D = \begin{bmatrix} C & C \end{bmatrix}$$

Exercise 2. Building Matrices in $\mathbb{R}^{2\times 3}$.

Give a numerical example of a matrix M in $\mathbb{R}^{2\times 3}$ with the following properties, when possible. When not, explain why such a matrix M cannot exist.

i) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ belong to $Col(M)$, and $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ belongs to $Ker(M)$.

ii) all the conditions above in i) as well as $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ belongs to $Ker(M)$

iii) $\dim Ker(M)$ is strictly smaller than than $rank(M)$

iv) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ belongs to $Col(M)$, and both $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ belong to $Ker(M)$.

Supplementary Exercises

From Gilbert Strang's Introduction to Linear Algebra,
Section 3.2 The Nullspace of A: Solving Ax = 0
Questions 9, 10, 18, 26, 27.