Homework 6

# Column Space and Kernel (or Nullspace).

$2 \times 2$ Matrices are transforms of $\mathbb{R}^2$

**Exercise 1. Geometric Transforms of $\mathbb{R}^2$.**

What are these matrices doing?

Try and sketch the transform of the unit cube (which is a square in $\mathbb{R}^2$):

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix}, \qquad B = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, \qquad P_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \qquad P_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

**Exercise 2. Dimension of the Kernel and the Image.**

What are the dimensions of the two main subspaces associated with the four matrices of Exercise 1, that is the column space and the kernel (or nullspace)?

$Col(X)$ the image of the whole space, or column space, and

$Ker(X)$, written $N(X)$ by G. Strang, a.k.a the nullspace.

Do not compute their sizes, doing Gaussian elimination, just watch at your sketches, and figure it out!

**Exercise 3. Products as geometric Transforms.**

First recall what is the meaning of the product of two square matrices, seen as transforms of $\mathbb{R}^n$:

$$BAx = y$$

Then answer the same question (size of the two main subspaces associated with the matrix) for the following products, without computing the products first, just sketching the transform and trying to figure out the column space and the kernel.

$$BA, \qquad AB, \qquad P_1 A, \qquad P_2^2, \qquad P_1 P_2$$